Worst-case Analysis of the Time-To-React Using Reachable Sets

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I. Risk assessment

Scope: Collision mitigation and collision avoidance systems in intelligent vehicles reduce the severity and number of accidents. To determine the optimal point in time at which such systems should intervene, time-based criticality metrics such as the Time-To-React (TTR) are commonly used.

Notation:

- t_0 is the initial time, t_f the final time, x_0 the initial state at t_0,
- $u(\cdot)$ is a input trajectory within the set of admissible inputs $\mathcal{U}$,
- $x\big(t; x_0, u(\cdot)\big)$ is the state at time t when applying $u(\cdot)$ starting at x_0,
- $\mathcal{F}(t)$ is the set of colliding states from a given obstacle prediction.

II. Time-To-React

Definition (Time-To-React) *The Time-To-React (TTR) is the maximum time we can continue the current trajectory $u_c(\cdot)$ before we have to execute an evasive trajectory $u(\cdot)$ to avoid entering the set of colliding states $\mathcal{F}(\cdot)$:*

$$\begin{aligned}
TTR := \sup_{t_*\in\mathbb{R}} \Big\{ t_* - t_0 \,\Big|\, & t_* \in [t_0, t_f], \exists u(\cdot) \in \mathcal{U}, \\
& \forall t \in [t_0, t_*] : x\big(t; x_0, u_c(\cdot)\big) \notin \mathcal{F}(t) \wedge \\
& \forall t \in [t_*, t_f] : x\big(t; x\big(t_*; x_0, u_c(\cdot)\big), u(\cdot)\big) \notin \mathcal{F}(t) \Big\}.
\end{aligned}$$

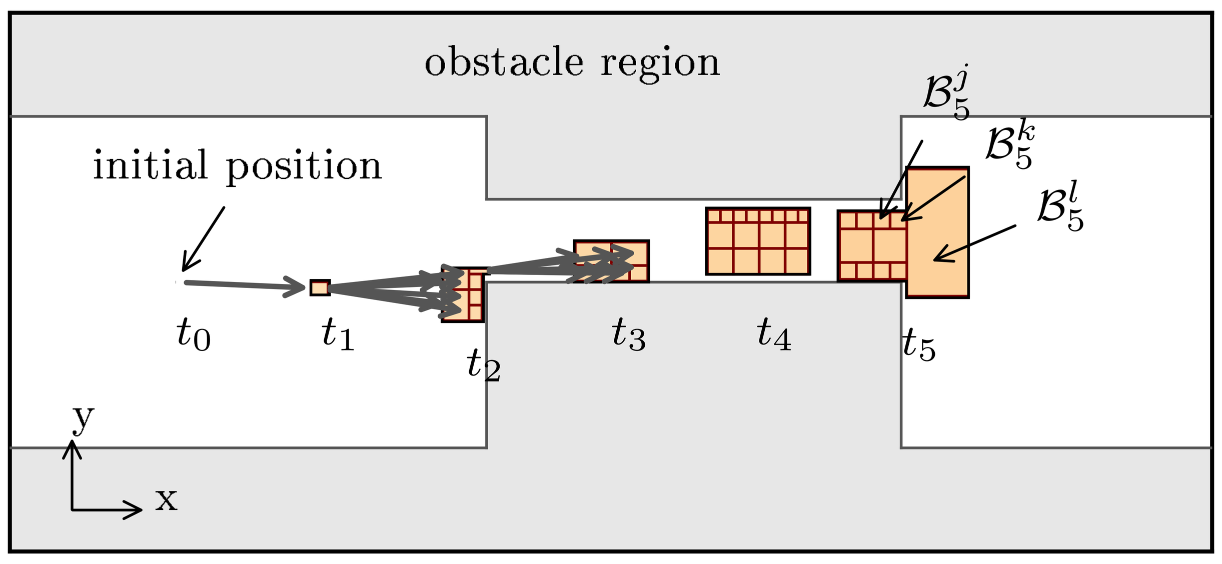
- Sampling-based trajectory planner can only evaluate a finite number of trajectories.
- To find the latest TTR, all possible evasive trajectories have to be considered.

III. Reachable set

The reachable set contains all possible trajectories:

Definition (Reachable set) *The reachable set is the set of states which are reachable at time t from an initial set $\mathcal{X}_0$ at time t_0 without entering $\mathcal{F}(\cdot)$:*

$$\begin{aligned}
\mathcal{R}(t; \mathcal{X}_0, t_0) := \Big\{ x\big(t; x_0, u(\cdot)\big) \,\Big|\, & x_0 \in \mathcal{X}_0, u(\cdot) \in \mathcal{U}, \\
& \forall \tau \in [t_0, t] : x\big(\tau; x_0, u(\cdot)\big) \notin \mathcal{F}(\tau) \Big\}.
\end{aligned}$$



IV. Worst-case analysis

We use over-approximative reachable sets to consider all possible evasive trajectories:

Proposition (Time-To-React using reachable sets) *The TTR is the last point in time along the current trajectory from which the reachable set is nonempty at the end of the planning horizon:*

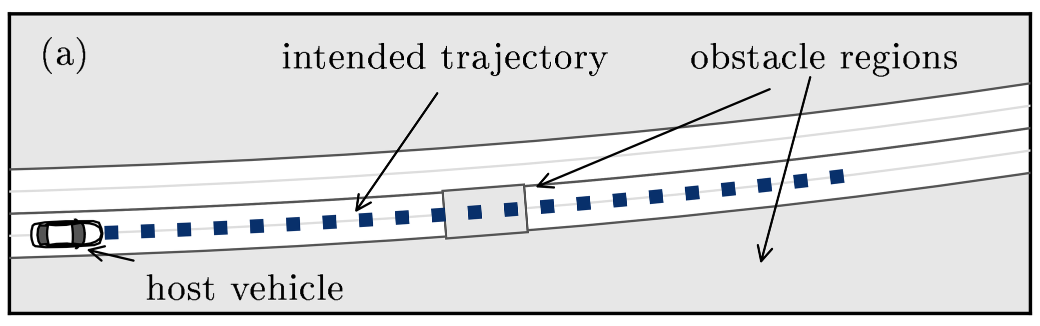
$$\begin{aligned}
TTR = \sup_{t_*\in\mathbb{R}} \Big\{ t_* - t_0 \,\Big|\, & t_* \in [t_0, t_f], \\
& \forall t \in [t_0, t_*] : x\big(t; x_0, u_c(\cdot)\big) \notin \mathcal{F}(t) \wedge \\
& \mathcal{R}\big(t_f; x\big(t_*; x_0, u_c(\cdot)\big), t_*\big) \neq \emptyset \Big\}.
\end{aligned}$$

Our worst-case TTR guarantees that no later evasive trajectory exists.

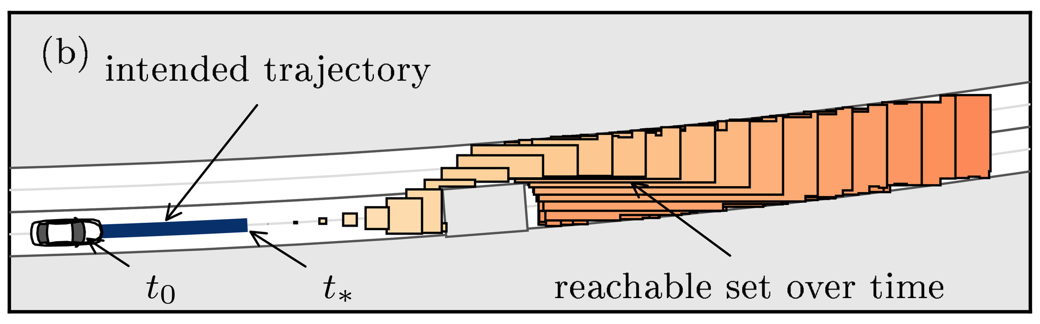
V. Numerical examples

Two-lane road (CommonRoad ID: S=Z_Overtake_1a)

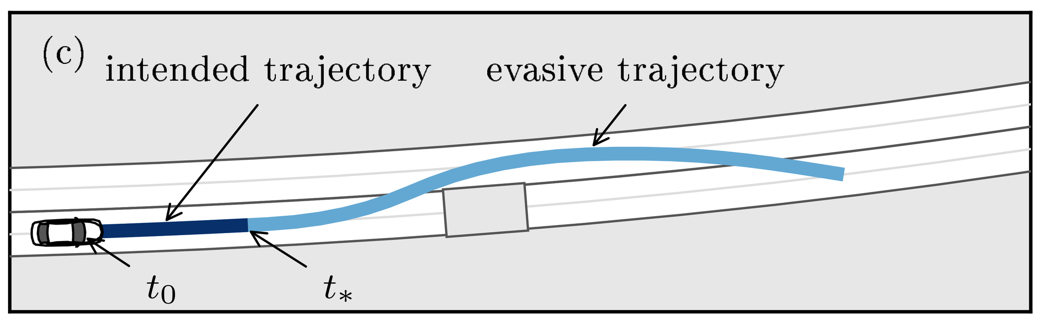
Initial configuration with intended trajectory $u_c(\cdot)$ and static obstacle regions:



The last nonempty reachable set starts at the intended trajectory at $t_* = 0.7\,\mathrm{s}$:

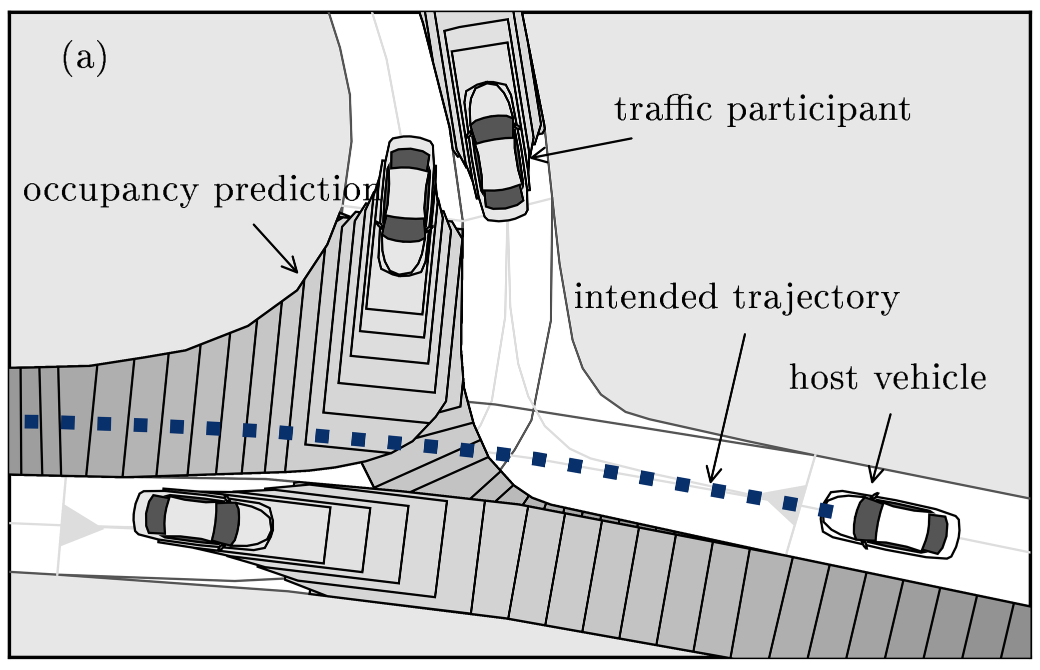


An estimate of the latest possible evasive trajectory also branches off at $t_* = 0.7\,\mathrm{s}$:

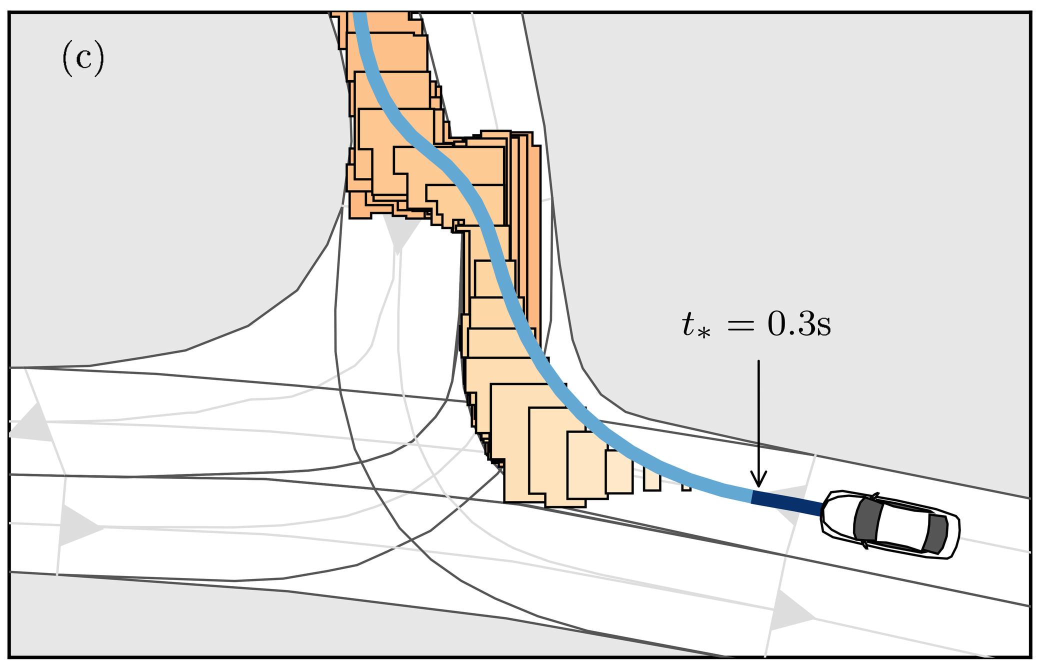


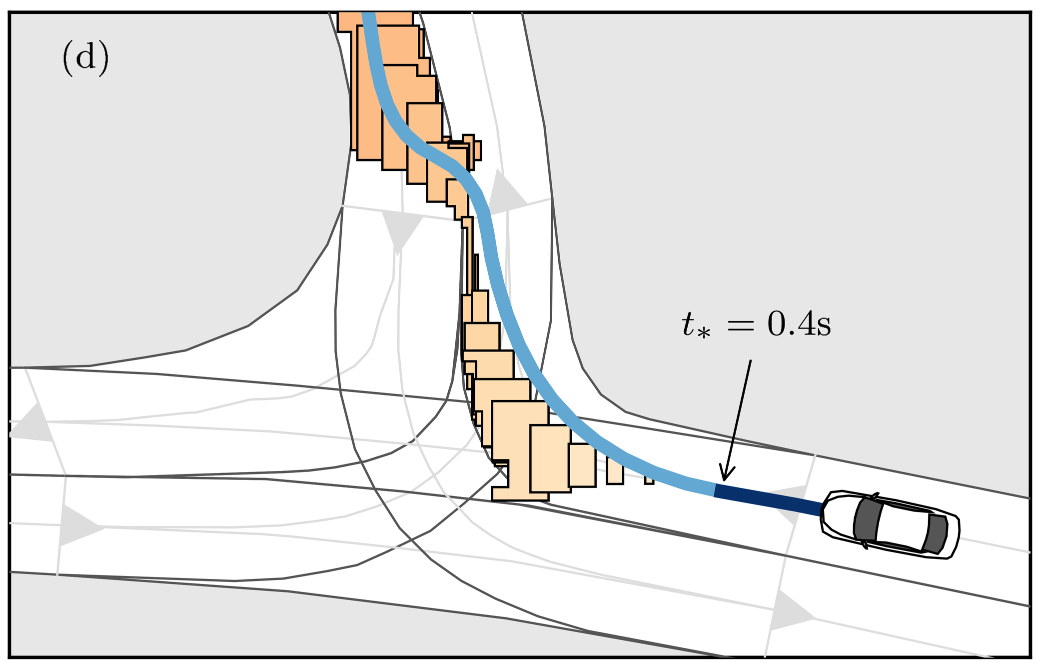
T-intersection (CommonRoad ID: S=GER_Ffb_2b)

Initial configuration with predicted occupancies:



The reachable set and an evasive trajectory starting at different TTR candidates t_*:





All our scenarios are available at **commonroad.in.tum.de**, which provides open-source benchmarks for trajectory planning.

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