I. Risk assessment

Scope: Collision mitigation and collision avoidance systems in intelligent vehicles reduce the severity and number of accidents. To determine the optimal point in time at which such systems should intervene, time-based criticality metrics such as the Time-To-React (TTR) are commonly used.

Notation:
- \( t_i \) is the initial time, \( t_f \) the final time, \( x_0 \) the initial state at \( t_0 \),
- \( u(\cdot) \) is an input trajectory within the set of admissible inputs \( U \),
- \( x(t; x_0, u(\cdot)) \) is the state at time \( t \) when applying \( u(\cdot) \) starting at \( x_0 \),
- \( \mathcal{F}(t) \) is the set of colliding states from a given obstacle prediction.

II. Time-To-React

Definition (Time-To-React) The Time-To-React (TTR) is the maximum time we can continue the current trajectory \( u_c(\cdot) \) before we have to execute an evasive trajectory \( u(\cdot) \) to avoid entering the set of colliding states \( \mathcal{F}(\cdot) \):

\[
TTR := \sup_{t_* \in \mathbb{R}} \left\{ t_* - t_0 \mid t_* \in [t_0, t_f], \exists u(\cdot) \in U, \forall t \in [t_0, t_*]: x(t; x_0, u(\cdot)) \notin \mathcal{F}(t) \land \forall t \in [t_*, t_f]: x(t; x_0, u(\cdot)), u(\cdot) \notin \mathcal{F}(t) \right\}.
\]

- Sampling-based trajectory planner can only evaluate a finite number of trajectories.
- To find the latest TTR, all possible evasive trajectories have to be considered.

III. Reachable set

The reachable set contains all possible trajectories:

Definition (Reachable set) The reachable set is the set of states which are reachable at time \( t \) from an initial set \( X_0 \) at time \( t_0 \) without entering \( \mathcal{F}(\cdot) \):

\[
\mathcal{R}(t; X_0, t_0) := \left\{ x(t; x_0, u(\cdot)) \mid x_0 \in X_0, u(\cdot) \in U, \forall \tau \in [t_0, t]: x(\tau; x_0, u(\cdot)) \notin \mathcal{F}(\tau) \right\}.
\]

IV. Worst-case analysis

We use over-approximative reachable sets to consider all possible evasive trajectories:

Proposition (Time-To-React using reachable sets) The TTR is the last point in time along the current trajectory from which the reachable set is nonempty at the end of the planning horizon:

\[
TTR = \sup_{t_* \in \mathbb{R}} \left\{ t_* - t_0 \mid t_* \in [t_0, t_f], \forall t \in [t_0, t_*]: x(t; x_0, u(\cdot)) \notin \mathcal{F}(t) \land \mathcal{R}(t_f; x(t_*, x_0, u(\cdot)), t_*) = \emptyset \right\}.
\]

Our worst-case TTR guarantees that no later evasive trajectory exists.

V. Numerical examples

Two-lane road (CommonRoad ID: S-Z_Overtake_1a)

Initial configuration with intended trajectory \( u_c(\cdot) \) and static obstacle regions:

The last nonempty reachable set starts at the intended trajectory at \( t_* = 0.7s \):

An estimate of the latest possible evasive trajectory also branches off at \( t_* = 0.7s \):

T-intersection (CommonRoad ID: S-GER_Ffb_2b)

Initial configuration with predicted occupancies:

The reachable set and an evasive trajectory starting at different TTR candidates \( t_* \):

All our scenarios are available at commonroad.in.tum.de, which provides open-source benchmarks for trajectory planning.

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